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Conditions for Failure of Normal Section in Flexural Reinforced Concrete Beams of Rectangular Cross-Section

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Abstract

The stress in longitudinal tensile reinforcement is one of the main important parameters while examining the technical state of under-reinforced concrete structures. The most important issue is to determine whether the external loads cause the close to yield stress in the main reinforcement. Yield stress in tensile reinforcement could be treated as the start of incipient failure of the flexural structure. The state of tensile reinforcement of flexural reinforced concrete structures could be examined by observing the properties of the normal cracks. The application of fracture mechanics of solids could be used for determining the actual damage to the structure by knowing only the measured height of the normal crack.

Keywords: concrete; crack, reinforcement; stress; defomation; deflection.

1. Introduction

The assessment of stress state in longitudinal tensile reinforcement is highly important while examining the technical state of under-reinforced concrete structures. The most important issue is to determine whether the external loads cause the close to yield stress in the main reinforcement. The appearance of yield stress in tensile reinforcement could be treated as the start of incipient failure of the flexural structure [1-4]. The propagation of cracks in flexural reinforced concrete beams is investigated extensively but such research usually is limited to the serviceability stage, i. e. before the failure starts [5-7]. Although it is also important to know the characteristics of the critical macro-crack which cause the actual failure of the member, e.g. critical depth of normal crack, which cause the yield stress in tensile reinforcement. The availability of such research data in scientific literature is limited [8].

The relationship between the depth of normal crack and stress state within the cross-section of the beam is proven by theoretical and experimental research many years ago [5], [9-12]. The stress in main reinforcement could be determined by using the data from experimental research, namely – the depth of normal crack which could be measured relatively easy in most cases. Thus the stress in tensile reinforcement could be determined for the examined beams without unloading.

2. Influence of the critical depth of the crack on the stress in the tensile reinforcement

Calculation model for the crack development (Fig. 1) is based on the rules provided by the fracture mechanics of solids [3], [13-16]. According to this theory, the two tips of each crack could be determined. One of them causes the propagation

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of the crack towards the neutral axis of flexural member. The position of the other tip coincide with the level of tensile reinforcement. The width of the crack tip which is close to the neutral axis is critical and generally govern the further crack development. The bond forces between concrete and reinforcing steel resist to the crack development.

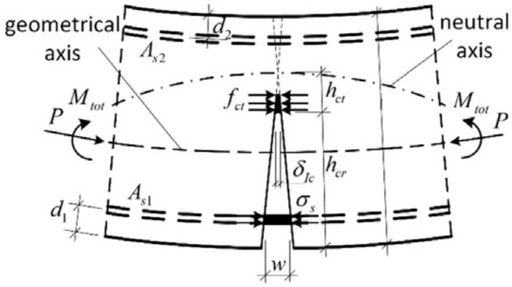

Fig. 1. The model for calculation of normal crack development

The parts of the member separated by the crack rotate around the point which is an intersection between crack plane and neutral axis. Distance between the crack surfaces within h_{cr} is proportional to the distances to neutral axis, see Fig. 1. The following formula could be written for calculation of stress in tensile reinforcement based on model shown in Fig. 1:

$$\sigma_s = 0.785\left[\frac{M_{tot}\left(0.75h_{cr}+h_{ct}\right)}{I_1}-\frac{P}{A_1}\right]\left(\frac{h_{cr}}{t}\right)^{\frac{1}{2}}\alpha \le \sigma_y, \tag{1}$$

where σ_s – actual stress in tensile reinforcement; $M_{tot} = M - Pe_{0p} - \Delta M$; M – bending moment; P– prestress force; e_{0p} – eccentricity of prestress force; $\Delta M = P(y - h_{cr} - h_{ct})$ – the increase of bending moment caused by matching the geometrical center and neutral axis in the design cross-section; y– distance from most tensile fibre of cross-section to geometrical axis (center of gravity); – depth of the crack; h_{ct} – height of the tensile zone above the crack; A_1 and I_1 – area of equivalent cross-section and second moment of area of equivalent cross-section respectively (when matching the geometrical center and neutral axis); b – width of cross-section; A_{S1} and A_{S2} – areas of tensile and compressive reinforcement respectively; a – dimensionless adjustment function which depends on the depth of the crack and geometry of the cross-section (usually the ratio $\frac{h_{cr}}{h}$); $t = \frac{A_{s1}}{b}$; σ_y – yield stress in tensile reinforcement.

When yield stress σ_y is reached in tensile reinforcement, the tensile zone of concrete above the crack is insignificant and may be neglected, i.e. $h_{ct} = 0$. Also because of significant plastic deformations in the tensile reinforcement the prestress force could be neglected too. Therefore Eq. (1) could be written in the following form:

$$\sigma_y \cong \left(\frac{0.59M_{u1}h_{cr,lim}}{I_1^*}\right)\left(\frac{h_{cr,lim}}{t}\right)^{\frac{1}{2}}\alpha, \tag{2}$$

here M_{u1} and $h_{cr,lim}$ – bending moment and critical depth of the crack respectively, when stress in tensile reinforcement reach its yield limit σ_y ($\sigma_{0.2}$ or $\sigma_{0.1}$).

The second moment of area of equivalent design cross-section could be calculated according to the following formula:

$$I_1^* = \frac{bh_1^3}{12} + bh_1\left(0.5h_1 - h_{cr}\right)^2 + 0.5\alpha_e A_{s1}h_{cr}^2 + \alpha_e A_{s2}\left(h_1 - h_{cr} - d_2\right)^2, \tag{3}$$

where the height of equivalent design cross-section $h_1 = -k_1 + \left(k_1^2 + k_2\right)^{\frac{1}{2}}$; $k_1 = \frac{(\alpha_e A_{s2} - h_{cr} b)}{b}$ (when $A_{S2} = 0$, $k_1 = -h_{cr}$); $k_2 = \frac{2\alpha_e (A_{s2} d_2 + A_{s2} h_{cr} - 0.5 A_{s1} h_{cr})}{b}$ (when $A_{S2} = 0$, $k_2 = \frac{\alpha_e A_{s1} h_{cr}}{b}$); depth of the crack $h_{cr} = h_{cr,lim}$; α_e – ratio between the modulus of elasticity of reinforcement and concrete; d_2 – distance from most compressive fibre of cross-section to the center of gravity of compressive reinforcement.

It is obvious that the precision of Eq. (2) directly depends on the adjustment function α. This function depend on many parameters of normal crack but the greatest influence is related to ratio $h_{cr} = h$ and reinforcement ratio ρ.

3. Influence of normal crack parameters on the adjustment function

The theory of relationship between the depth of normal cracks and stress state in cross-section is based on numerous experimental research data [11], [15]. This theory allows considering the case when the slip between tensile reinforcement and concrete occurs after the crack appearance, i.e. hypothesis of plane sections is not valid. In such a case the deformations in concrete and reinforcement are not equal anymore because of damaged bond between them. Thus the stress in tensile reinforcement could be calculated by using the following equation system:

$$\left.\begin{aligned} h_{ct} &= \frac{\left[\left(I - S h_{cr}\right) - \left(S - A h_{cr}\right) d_1 - \frac{b h_{ct}^3}{12}\right]}{\left[\frac{M}{2 f_{ct}} + S - \left(\frac{P}{2 f_{ct}} + A\right) d_1\right]} \\ \sigma_s &= \frac{2 f_{ct}\left[\frac{\left(S - A h_{cr}\right)}{h_{ct}} - \frac{P}{2 f_{ct}} - A\right]}{A_{s1}} \end{aligned}\right\}, \tag{4}$$

Where A, S, I and M – respectively area of cross-section, first moment of area, second moment of area and bending moment in respect to the edge of cross-section subjected to the greatest tension. The area of cross-section within the depth of the crack (including area of tensile reinforcement A_{S1}) is neglected.

The values of the stress in tensile reinforcement calculated according to Eq. (4) were similar to the ones obtained from the experimental research on flexural beams of rectangular and tee cross-sections, when beams were loaded by 40 to 80% of the ultimate load [5], [12], [17-18]..

When calculating the stress in reinforcement according to Eqs. (4) the use of expression is avoided and thus these equations are suitable for determination of adjustment factor α itself. The stress in tensile reinforcement σ_S could be calculated by using Eqs. (4) and relationship between h_{cr} and M obtained from experimental research. Then adjustment function could be calculated by putting M, h_{cr} and σ_S values (determined according to Eqs (4)) to Eq. (2) and assuming that $h_{ct} = 0$. On the next stage the influence of ratio h_{cr} / h and other parameters on adjustment function α could be determined.

When stress in tensile reinforcement is close to yield state the strength of compressive concrete remains partially unused. Thus in such stress state the triangular design diagram of stress distribution within the compressive zone is most relevant.

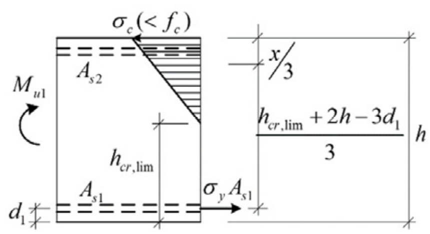

Fig. 2. Design state of stress within the cross-section when tensile reinforcement yields

When performance of tensile concrete above the crack is neglected – carrying capacity of the beam increases. On the other hand, such increase of carrying capacity should be reduced because of ignorance of the compressive reinforcement and plastic deformations in compressive concrete. When taking into account these assumptions (Fig. 2) and the condition of static equilibrium between the moments of internal and external forces, we can write the following expression for calculation of carrying capacity of the beam:

$$M_{u1} \cong \frac{\sigma_y A_{s1}(h_{cr,lim} + 2h - 3d_1)}{3}. \tag{5}$$

When calculating the stress σ_y, Equations (2), (4) and (5) results the same values because of insignificant influence of the tensile concrete above the crack on the stress state.

The data of experimental research on 28 beams of rectangular cross-section was used to analyze the adjustment function α. In 26 of these experimental beams the various pre-stress degree and low reinforcing ratio was present. Reinforcing ratio in the remaining 2 beams was significantly higher [17-18]. The main parameters of experimental beams are given in Table 1.

All tested beams failed in pure bending zone which was middle one third of the beam span. The span for all beams was 1.80 m with exception of two S group beams which span was 1.20 m. One beam in each series (including beams S1 and S2) was loaded in steps 0.1 M_{u2} all the way to the incipient failure. The remaining test beams were loaded in steps 0.1 M_{u2} until the (1.3 – 1.5)M_{cr} (here M_{u2} and M_{cr} – ultimate and cracking moments of the beams respectively) and then unloaded. Later the beams were loaded until the (1.75 – 2.1)M_{cr} and unloaded again. Finally the beams were loaded in steps 0.2 M_{u2} until the incipient failure. The depths of the cracks within the pure bending zone were measured by 24 times magnifying microscope. Depth of one normal crack at the concrete failure point in compressive zone was additionally monitored by measuring longitudinal deformations. The duration of each loading step was 20 to 30 minutes.

Table 1. Main parameters of experimental beams

Test beam group	Quantity	Dimensions of cross-section $b \times h$, mm	Concrete strength $f_{c,cube}$, MPa	Characteristics of tensile reinforcement			
				ρ, %	P, kN	Quantity of rebars and their diameter, mm	σ_y, MPa
A	4		53.4		1325		
B	4		50.0	0.79	776	6∅5 (hard wire)	
C	2		54.0		0		1160
D	4	100 × 180	48.5	0.37	677	3∅5	
AI	4		40.5		750		
BI	4		52.0	0.94	473	3∅8 (deformed rebars)	587
CI	4		55.0		0		
S1	1	100 × 195	37.2	1.82	0	4∅10 (deformed rebars)	477
S2	1				0	4∅10 (even rebars)	291

The values of adjustment function α were determined by using the technique described above and the relationships between the depths of normal cracks and bending moments. Also the clear influence of the ratio $h_{cr,lim}/h$ and reinforcing ratio on the adjustment function α (when $\sigma_s = \sigma_y$) was determined. The relationship between the measured depth of the crack $h_{cr,lim}$ and depth h_{ct} obtained from Eq. (4), when $\sigma_s = \sigma_y$, was determined by analyzing experimental research data, see Table 2.

According to the analysis the adjustment function α could be calculated by the following formula (coefficient of correlation 0.9933):

$$\alpha = \frac{\dfrac{15.53 h_{cr}}{h} - 1.41}{\rho\psi}, \tag{6}$$

where $\rho = A_{s1} / (bd)100$ – reinforcing ratio; d – design depth of cross-section; factor $\psi = 1.6$ when $\rho = 0.37\%$, $\psi = 1$ when $\rho = (0.79 – 1.0)\%$ and $\psi = 0.65$ when $\rho = 1.82\%$. The intermediate values of product $\rho\psi$ could be obtained by interpolating.

Table 2. Relation between adjustment function α and geometrical characteristics of the beams

Test beam group	Test beam code	h_{cr}	h_{ct}	α	$\frac{h_{cr}}{h}$	$\frac{h_{ct}}{h}$
A	A-1	99.00	9.84	7.22	0.054	0.055
	A-2	104.40	8.93	7.47	0.049	0.050
	A-3	86.40	12.58	6.02	0.069	0.070
	A-4	88.20	12.21	6.28	0.067	0.068
B	B-1	122.40	5.29	9.18	0.029	0.029
	B-2	129.60	4.19	9.74	0.023	0.023
	B-3	127.80	4.37	9.56	0.024	0.024
	B-4	124.20	5.10	9.25	0.028	0.028
C	C-3	145.80	2.37	11.04	0.013	0.013
	C-4	147.60	2.37	11.04	0.013	0.013
D	D-1	91.80	24.00	6.25	0.039	0.133
	D-2	108.00	15.34	7.68	0.025	0.085
	D-3	99.00	20.76	6.92	0.033	0.115
	D-4	81.00	28.12	5.47	0.045	0.156
AI	AI-1	100.80	11.70	7.37	0.065	0.065
	AI-2	86.40	15.84	6.18	0.088	0.088
	AI-3	90.00	14.58	6.41	0.081	0.081
	AI-4	82.80	16.92	5.74	0.094	0.094
BI	BI-1	117.00	8.82	8.76	0.049	0.049
	BI-2	124.20	7.20	9.36	0.040	0.040
	BI-3	117.00	8.82	8.76	0.049	0.049
	BI-4	113.40	9.72	8.45	0.054	0.054
CI	CI-1	140.40	4.14	10.73	0.023	0.023
	CI-2	138.60	4.68	10.5	0.026	0.026
	CI-3	120.60	8.46	9.04	0.047	0.047
	CI-4	133.20	5.40	10.15	0.030	0.030
S1	S-1	134.60	4.88	9.51	0.038	0.025
S2	S-2	138.50	6.44	9.98	0.042	0.033

The height of tensile concrete zone above the crack could be calculated by the following formula (coefficient of correlation 0.9616):

$$h_{ct} = \frac{\left(1.84 - 2.12\frac{h_{cr}}{h}\right)h}{10\eta\omega}, \tag{7}$$

where $\eta = 1$, when deformed bars are used for reinforcing and $\eta = 1.2$, when deformed wires are used; $\omega = 125\rho$, when $\rho < 0.8\%$ and $\omega = 1$, when $\rho \geq 0.8\%$.

According to the criteria of crack propagation known in fracture mechanics the following empirical relationship between parameters of normal crack could be written [14]:

$$w = \frac{\delta_{Ic} h_{cr}}{h_{ct}}. \quad (8)$$

Here critical width of crack tip $\delta_{Ic} = 0.00012 d_1 \sqrt[3]{\varnothing}$ ($\varnothing$ – diameter of tensile reinforcement). Eqs. (7) and (8) could be used to calculate the depth of the crack:

$$h_{cr} = \frac{0.18hw}{0.21w + \delta_c \eta \omega}. \quad (9)$$

Such theoretical-empirical expression assist the more reliable control of measurements of normal crack parameters when investigating the structures.

4. Conclusions

1. The possibilities offered in fracture mechanics Eqs. (1) and (2) could be used for analysis of stress state in flexural reinforced concrete members together with known section method when writing the equations of static equilibrium between internal and external forces Eqs. (4) and (5).
2. Adjustment function α allows the evaluation of geometrical characteristics of reinforced concrete member. Eq. (6) is valid only for the tested beams described in this paper. It should be refined for the beams of different cross-section shape and with different (especially – higher) reinforcing ratio. Adjustment function could be refined by either using the method presented in this paper or directly by experimental research. It is not enough to know the ratio h_{cr} / h – the reinforcing ratio should be estimated also.
3. The formulas presented in this paper (e.g. Eq. (9)) allow the accurate enough representation of relationship between the various parameters of normal crack of flexural beam. It also allow the more reliable estimation of actual state of flexural beam during the on-site investigation.

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